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Correlator Compensation Requirements for Passive Time-Delay Estimation with Moving Source or Receivers

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Abstract—An analysis is given of the effects of source or receiver motions on the output of a cross correlator used to estimate the source time-delay difference across a baseline. For both narrow-band and wide-band correlators, the need for time-varying correlator delay compensation is quantified for the case of a quadratic signal delay-difference variation. Two useful concepts which emerge are 1) the three-dimensional delay/delay-rate/delay-acceleration mean ambiguity function of the source signal, and 2) the equivalent r -domain filter, which transforms the source autocorrelation function into the mean output of the mismatched correlator.

The required correlator compensation is approximately a quadratic delay modulation matched to the input delay-difference function. For 3 dB peak output loss with a narrow-band signal, the maximum allowable delay-rate mismatch will produce 158° of phase rotation at the centroid frequency f_0 during the correlation integration time T , while the maximum allowable delay-acceleration mismatch will produce 156° of quadratic phase rotation at f_0 between the center ($t = 0$) and each edge ($t = \pm T/2$) of the correlator integration window. For broad-band signals, the mismatch tolerances become about 11 percent tighter.

I. INTRODUCTION

GIVEN two sensors separated by a baseline, passive estimation of the time-delay difference between signals received from a common source may be accomplished by cross correlating the appropriately filtered [1] sensor outputs.

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When source or receiver motions cause the delay difference to vary, during the correlator integration time T , by more than the correlation time width of the source signal, then the output correlogram is degraded (smeared) unless the correlator implements a compensating delay modulation during T . In many cases the simple expedient of reducing T causes an unacceptable reduction in processing gain.

For a linear delay-difference variation during T , the optimum correlator delay modulation is also linear in time during T , so that one receiver output is time scaled (compressed or expanded) by a constant factor before cross correlating [2]. For a narrow-band signal, this compensation can be approximated by a constant frequency shift [3], [6], [7].

For a quadratic delay-difference variation during T , the optimum correlator delay modulation is approximately quadratic in time during T , so that one receiver output is time scaled by a linearly varying factor before cross correlating. For a narrow-band signal, this compensation can be approximated by a linearly varying frequency shift [3]. If the correlator can only implement a linear delay modulation (or its narrow-band approximation by a constant frequency shift), then the maximum allowable value of T for a specified correlogram degradation becomes a function of the geometry and motion scenario, together with the centroid and width of the source spectrum [3], [4].

All these effects for either broad-band or narrow-band correlators are treated in a unified manner by employing analytic-signal representation, assuming a stochastic source emission,

and examining the mean (ensemble average) correlogram output as a function of errors (mismatches) in the correlator delay-modulation schedule. Correlator compensation requirements, for a specified degradation of 3 dB in the mean peak correlator output, are then obtained as error budgets on the allowable mismatches in compensating for delay-difference rates and accelerations (which at any reference frequency, and in particular at the centroid frequency f_0 of the source spectrum, may be translated into allowable values of frequency difference and frequency-difference rate).

II. MEAN CORRELATOR OUTPUT WITH QUADRATIC DELAY VARIATIONS

A. General Approach

Assume the source emits a waveform $s(t)$, which is a sample function of a zero-mean stationary random process with autocorrelation function (acf)

$$R_s(\tau) = E_s\{s(t)s(t+\tau)\}. \quad (1)$$

Neglecting medium distortion and attenuation, the signal received at sensor " k " ($k=1, 2$) is delay-modulated because of source or receiver motion

$$s_k(t) = s[t - \tau_k(t)]. \quad (2)$$

If the propagation delays $\tau_k(t)$ are assumed to vary quadratically with time during the observation interval T_{obs} ,

$$\tau_k(t) = \tau_{ok} + \tau'_{ok}t + \tau''_{ok}t^2/2, \quad |t| \leq T_{\text{obs}}/2, \quad (3)$$

then the delay difference is also quadratic in time

$$\begin{aligned} \Delta(t) &= \tau_1(t) - \tau_2(t) \\ &= \Delta_0 + \Delta'_0 t + \Delta''_0 t^2/2 \end{aligned} \quad (4)$$

where

$$\begin{aligned} \Delta_0 &= \tau_{01} - \tau_{02} \\ \Delta'_0 &= \tau'_{01} - \tau'_{02} \\ \Delta''_0 &= \tau''_{01} - \tau''_{02}. \end{aligned} \quad (5)$$

The cross correlator must then implement a time-varying delay-modulation schedule during its integration time T to compensate for the moving correlation point. We assume a quadratic¹ correlator delay-modulation schedule given by

$$\Delta_c(t) = \tau_c + \lambda_c t + \alpha_c t^2/2, \quad |t| \leq T/2 \quad (6)$$

so that when the correlator inputs, including additive noises, are

$$\begin{aligned} x(t) &= s[t - \tau_1(t)] + n_1(t) \\ y(t) &= s[t - \tau_2(t)] + n_2(t), \end{aligned} \quad (7)$$

then the correlator output is

$$F = F(\tau_c, \lambda_c, \alpha_c) = \frac{1}{T} \int_{-T/2}^{T/2} x(t)y[t - \Delta_c(t)] dt. \quad (8)$$

¹This delay schedule can give perfect compensation for a constant or linearly-varying delay difference during T , but can only give a good approximation to perfect compensation when $\tau''_{02} \neq 0$ (Appendix B).

The correlator attempts to compensate for Δ'_0 and Δ''_0 by a proper choice of λ_c and α_c , after which it performs (e.g., by parallel processing) a scan in τ_c to find the delay τ_{cm} at which F is a maximum. The value τ_{cm} is its estimate of the delay difference Δ_0 between the received signals at the instant $t=0$ (taken as the center of its integration window).

Since F is a random function of τ_c because s, n_1 and n_2 are sample functions of random processes (assumed independent), we require a large product ($T \cdot B_{\text{signal}}$) so that the time average performed by the correlator gives a useful approximation to the ensemble average over the noise and signal distributions, i.e., so that F versus τ_c is a good sampling image of the signal acf with a delay shift. The correlator compensation requirements may then be determined by studying the mean (ensemble average) correlator output [5], [6] $R_d = E_{s, n_1, n_2}\{F\}$ as a function of the mismatches in choosing τ_c, λ_c , and α_c . Combining (7) and (8) and averaging over n_1 and n_2 yields $R_d = E_s\{F | n_1 = n_2 = 0\}$, which expresses the well-known [5], [11] result that additive zero-mean independent noises do not affect the *mean* cross-correlator output. Thus for our purposes the additive noise will be ignored.

B. Analytic-Signal (Preenvelope) Notation

In discussing both wide-band and narrow-band correlators it is convenient to use analytic-signal or preenvelope notation [6], [8], [9], where (\sim) denotes $[(\cdot) + j(\hat{\cdot})]$ with $(\hat{\cdot})$ the Hilbert transform of (\cdot) .

Thus the real source waveform is

$$s(t) = \text{Re}[\tilde{s}(t)] \quad (9)$$

where

$$\tilde{s}(t) = s(t) + j\hat{s}(t) \quad (10)$$

and the complex acf of $\tilde{s}(t)$ may be shown to be [6], [8], [9]

$$\begin{aligned} R_{\tilde{s}}(\tau) &= E_{\tilde{s}}\{\tilde{s}^*(t)\tilde{s}(t+\tau)\} \\ &= 2\tilde{R}_s(\tau) = 2[R_s(\tau) + j\hat{R}_s(\tau)] \end{aligned} \quad (11)$$

which is twice the preenvelope of $R_s(\tau)$.

With a source mean power density spectrum $W_s(f)$, then

$$R_s(\tau) = \int_{-\infty}^{\infty} W_s(f) e^{j2\pi f\tau} df \quad (12)$$

and [8]

$$\tilde{R}_s(\tau) = 2 \int_0^{\infty} W_s(f) e^{j2\pi f\tau} df. \quad (13)$$

C. Mean Output of Delay-Modulated Correlator

Using analytic-signal notation, the basic expression for the mean correlator output \tilde{R}_d is now derived. Referring to the functional diagram shown in Fig. 1, the analytic source signal $\tilde{s}(t)$ experiences the time-varying propagation delays $\tau_k(t)$, $k=1, 2$, and appears at correlator input port " k " as $\tilde{s}_k(t) = \tilde{s}(t - \tau_k(t))$. The correlator imposes the compensating delay modulation $\Delta_c(t)$ on $\tilde{s}_2(t)$ to yield $\tilde{s}_3(t) = \tilde{s}_2[t - \Delta_c(t)] = \tilde{s}\{t - \Delta_c(t) - \tau_2[t - \Delta_c(t)]\}$, and then evaluates the time

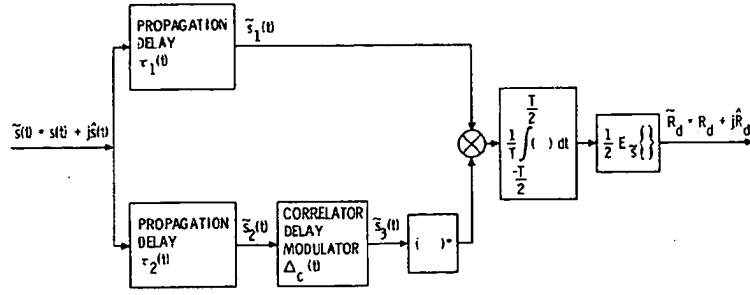


Fig. 1. Schematic for mean output of delay-modulated correlator.

average over $(-T/2, T/2)$ of the product $\tilde{s}_1(t) \tilde{s}_3^*(t)$. The mean correlator output \tilde{R}_d is then half the ensemble average (over \tilde{s}) of this finite time average, which from (11) is seen to be

$$\begin{aligned} \tilde{R}_d &= \frac{1}{2T} \int_{-T/2}^{T/2} E_{\tilde{s}} \{ \tilde{s}^*(t - \Delta_c(t) - \tau_2[t - \Delta_c(t)]) \\ &\quad \cdot \tilde{s}[t - \tau_1(t)] \} dt \\ &= \frac{1}{2T} \int_{-T/2}^{T/2} R_{\tilde{s}}[\epsilon(t)] dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} \tilde{R}_s[\epsilon(t)] dt \end{aligned} \quad (14)$$

where

$$\epsilon(t) = \Delta_c(t) - \{\tau_1(t) - \tau_2[t - \Delta_c(t)]\} \quad (15)$$

is the negative of the delay difference at the input to the multiplier.

Evaluating (15) with (3) and (6), we obtain after some minor approximations (Appendix B)

$$\begin{aligned} \epsilon(t) &\approx (\beta_2 \tau_c - \Delta_0) + (\beta_2 \lambda_c - \Delta'_0) t + (\beta_2 \alpha_c - \Delta''_0) t^2/2 \\ &= \beta_2(\tau + \lambda t + \alpha t^2/2) \end{aligned} \quad (16)$$

where we define

$$\begin{aligned} \beta_2 &= 1 - \tau'_{02} \\ \tau &= \tau_c - \Delta_0/\beta_2 \\ \lambda &= \lambda_c - \Delta'_0/\beta_2 \\ \alpha &= \alpha_c - \Delta''_0/\beta_2 \end{aligned} \quad (17)$$

so that (14) is

$$\tilde{R}_d = \tilde{R}_d(\tau, \lambda, \alpha) = \frac{1}{T} \int_{-T/2}^{T/2} \tilde{R}_s[\beta_2(\tau + \lambda t + \alpha t^2/2)] dt, \quad (18)$$

which may be interpreted [7] as a three-dimensional mean ambiguity function of the source waveform. For our purposes it is simply the mean pre-envelope of the correlator output, as a function of correlator mismatches in delay (τ), delay rate (λ), and delay acceleration (α). With perfect compensation in delay rate and delay acceleration, $\lambda = \alpha = 0$ so that

$$\tilde{R}_d(\tau, 0, 0) = \tilde{R}_s(\beta_2 \tau). \quad (19)$$

For a wide-band correlator, the mean real output is given by the real part of $\tilde{R}_d(\tau, \lambda, \alpha)$ from (18). For a narrow-band correlator this output versus τ has the form of a modulated carrier, and since the delay as measured by the "carrier" phase is too ambiguous to be useful, we rely instead on the modulated source bandwidth for delay determination [7] by finding that τ_{cm} which maximizes the envelope $|\tilde{R}_d(\tau, \lambda, \alpha)|$.

D. Equivalent Filter for Mismatched Correlator

A useful concept in understanding the effects of correlator mismatches on the mean output \tilde{R}_d is that of the equivalent filter [5]. If we put (13) in (18), interchange the order of integration, and change variable

$$\beta_2 f \rightarrow f, \quad (20)$$

then we obtain

$$\tilde{R}_d(\tau, \lambda, \alpha) = \int_0^\infty \left[\frac{2}{\beta_2} W_s \left(\frac{f}{\beta_2} \right) \right] H_d(f, \lambda, \alpha) e^{j2\pi f \tau} df \quad (21)$$

where

$$H_d(f, \lambda, \alpha) = \frac{1}{T} \int_{-T/2}^{T/2} e^{j2\pi f(\lambda t + \alpha t^2/2)} dt. \quad (22)$$

The bracketed term in (21) is the Fourier transform of $\tilde{R}_s(\beta_2 \tau)$, and $H_d(f, \lambda, \alpha)$ is an equivalent filter in the following sense: with τ analogous to a time variable, then $\tilde{R}_d(\tau, \lambda, \alpha)$ considered as a function of τ is the result of passing the "time" waveform $\tilde{R}_s(\beta_2 \tau)$ through a filter with frequency response $H_d(f, \lambda, \alpha)$ and τ -domain impulse response

$$h_d(\tau, \lambda, \alpha) = \int_{-\infty}^\infty H_d(f, \lambda, \alpha) e^{j2\pi f \tau} df. \quad (23)$$

Note that H_d is *not* a physical filter, but rather a mathematical analogy, as indicated by Fig. 2.

By completing the square in the exponent in (22), we obtain

$$\begin{aligned} H_d(f, \lambda, \alpha) &= \frac{e^{-j\pi(\lambda^2 f/\alpha)}}{2x} \left\{ Z \left[\left(1 + \frac{2\lambda}{\alpha T} \right) x \right] \right. \\ &\quad \left. + Z \left[\left(1 - \frac{2\lambda}{\alpha T} \right) x \right] \right\} \end{aligned} \quad (24)$$

where

$$x = T\sqrt{f\alpha/2}$$

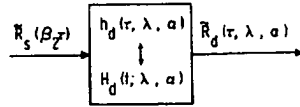


Fig. 2. Equivalent filter for delay-modulated correlator.

and

$$Z(u) = C(u) + jS(u) = \int_0^u e^{j(\pi/2)t^2} dt \quad (25)$$

is the complex Fresnel integral, obeying

$$\begin{aligned} Z(-u) &= -Z(u) \\ Z(ju) &= jZ^*(u). \end{aligned} \quad (26)$$

For special cases we obtain directly from (22),

$$H_d(f, 0, 0) = 1 \quad (27)$$

$$H_d(f, \lambda, 0) = \text{sinc}(\lambda f T),$$

$$\text{sinc}(u) = \frac{\sin \pi u}{\pi u}, \quad (28)$$

while from (24) we obtain

$$H_d(f, 0, \alpha) = \frac{Z(x)}{x}. \quad (29)$$

The corresponding τ -domain impulse response $h_d(\tau, \lambda, \alpha)$ is obtained by inverting (22), and differs according to whether $\lambda \geq \alpha T/2$:

$$h_d(\tau, \lambda, \alpha) \Big|_{\lambda > \alpha T/2} = \begin{cases} \frac{1}{T\sqrt{\lambda^2 - 2\alpha\tau}}, \\ 0, \end{cases}$$

and

$$h_d(\tau, \lambda, \alpha) \Big|_{\lambda < \alpha T/2} = h_d(\tau, \lambda, \alpha) \Big|_{\lambda > \alpha T/2} + h_2(\tau, \lambda, \alpha)$$

where

$$h_2(\tau, \lambda, \alpha) = \begin{cases} \frac{2}{T\sqrt{\lambda^2 - 2\alpha\tau}}, & \left(\frac{-\alpha T^2}{8} + \frac{\lambda T}{2}\right) \leq \tau < \frac{\lambda^2}{2\alpha} \\ 0, & \text{otherwise.} \end{cases} \quad (31)$$

For special cases this reduces to

$$h_d(\tau, 0, 0) = \delta(\tau) \quad (32)$$

$$h_d(\tau, \lambda, 0) = \begin{cases} \frac{1}{\lambda T}, & |\tau| \leq \frac{\lambda T}{2} \\ 0, & \text{otherwise} \end{cases} \quad (33)$$

and

$$h_d(\tau, 0, \alpha) = \begin{cases} \frac{2}{T\sqrt{-2\alpha\tau}}, & \frac{-\alpha T^2}{8} \leq \tau < 0 \\ 0, & \text{otherwise.} \end{cases} \quad (34)$$

III. CORRELATOR COMPENSATION REQUIREMENTS FOR DELAY-DIFFERENCE RATES AND ACCELERATIONS

A. General Approach

For both narrow-band and broad-band source spectra, we consider peak loss, resolution, and possible bias in the mean correlator output as a function of mismatches in the correlator delay-compensation parameters. Most of the results are well known when expressed in terms of range variation, rather than (as here) delay-variation coefficients. For physical interpretation and comparisons, we assume that the range from the source to sensor " k " ($k = 1, 2$) varies quadratically during the observation interval

$$r_k(t) = r_{0k} + r'_{0k}t + r''_{0k}t^2/2, \quad |t| \leq T_{\text{obs}}/2. \quad (35)$$

Thus, to obtain consistency with our model (3) of quadratic time variation of the propagation delays $\tau_k(t)$, we use the results given in Appendix A to obtain the correspondences shown in Table I.

B. Delay-Rate Mismatch with Perfect Delay-Acceleration Match

1) *Derivation of Results:* From (18) with $\alpha = 0$,

$$\begin{aligned} \tilde{R}_d(\tau, \lambda, 0) &= \frac{1}{T} \int_{-T/2}^{T/2} \tilde{R}_s[\beta_2(\tau + \lambda t)] dt \\ &= \frac{1}{\beta_2 \lambda T} \int_{\beta_2(\tau - (\lambda T/2))}^{\beta_2(\tau + (\lambda T/2))} \tilde{R}_s(x) dx \end{aligned} \quad (36)$$

$$\begin{aligned} \left(\frac{-\alpha T^2}{8} - \frac{\lambda T}{2}\right) \leq \tau \leq \left(\frac{-\alpha T^2}{8} + \frac{\lambda T}{2}\right) \\ \text{otherwise} \end{aligned} \quad (30)$$

with an equivalent expression given by (21) and (28) as

$$\tilde{R}_d(\tau, \lambda, 0) = \int_0^\infty \frac{2}{\beta_2} W_s\left(\frac{f}{\beta_2}\right) \text{sinc}(\lambda T f) e^{j2\pi f \tau} df. \quad (37)$$

The frequency functions pertinent to (37) are shown in Fig. 3 for a block source spectrum of width B centered at f_0 , normalized so that

$$\tilde{R}_s(\tau) = \text{sinc}(B\tau) e^{j2\pi f_0 \tau}. \quad (38)$$

Since $H_d(f, \lambda, 0)$ is purely real, then (refer to Fig. 2) by the principle of stationary phase [12] the τ -centroid of $\tilde{R}_d(\tau, \lambda, 0)$ is the same as that of $\tilde{R}_s(\beta_2\tau)$, which is at $\tau = 0$. Thus $H_d(f, \lambda, 0)$ introduces no τ -bias which varies with λ , and we need consider only the main-axis responses $\tilde{R}_d(\tau, 0, 0)$ and $\tilde{R}_d(0, \lambda, 0)$.

From (36) and (38),

$$\tilde{R}_d(\tau, 0, 0) = \tilde{R}_s(\beta_2\tau) = \text{sinc}(\beta_2 B\tau) e^{j2\pi f_0 \beta_2 \tau}, \quad (39)$$

while from (37) and Fig. 3,

TABLE I
RELATIONS BETWEEN DELAY AND RANGE COEFFICIENTS

ITEM	KINEMATIC EQUIVALENT	
	MOVING SOURCE, FIXED RECEIVERS (*)	FIXED SOURCE, MOVING RECEIVERS
$\beta_k - 1 - \tau'_{0k}/c$, $k=1,2$	$1/(1 + r'_{0k}/c)$	$1 - r'_{0k}/c$
τ_{0k}	$\beta_k r'_{0k}/c$	r'_{0k}/c
$\Delta_0 = \tau_{01} - \tau_{02}$	$(\beta_1 r'_{01} - \beta_2 r'_{02})/c$	$(r'_{01} - r'_{02})/c$
$\Delta'_0 = \tau'_{01} - \tau'_{02}$	$\beta_2 - \beta_1$	$\beta_2 - \beta_1$
$\Delta''_0 = \tau''_{01} - \tau''_{02}$	$(\beta_1^3 r'_{01} - \beta_2^3 r'_{02})/c$	$(r'_{01} - r'_{02})/c$

(*) EXACT FOR $r'_{0k} \rightarrow 0$.

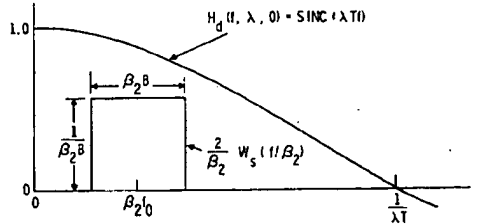


Fig. 3. Scaled source spectrum and equivalent filter response for delay-rate mismatch λ and correlator integration time T .

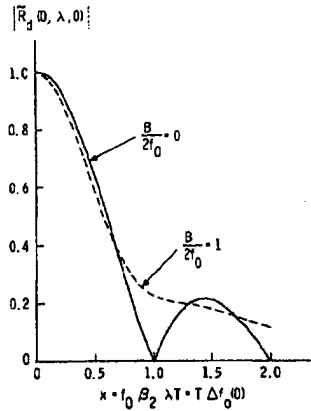


Fig. 4. Delay-rate mismatch response at $\tau = 0$ (block source spectrum, bandwidth B , centroid f_0).

$$\begin{aligned} \tilde{R}_d(0, \lambda, 0) &= \frac{1}{\beta_2 B} \int_{\beta_2(f_0 - B/2)}^{\beta_2(f_0 + B/2)} \text{sinc}(\lambda T f) df \\ &= \frac{\text{Si}[\pi x(1 + \mu)] - \text{Si}[\pi x(1 - \mu)]}{2\pi x \mu} \end{aligned}$$

where

$$\begin{aligned} \mu &= \frac{B}{2f_0} \\ x &= \beta_2 f_0 \lambda T \end{aligned}$$

$$\text{Si}(x) = \int_0^x \frac{\sin t}{t} dt.$$

For a source bandwidth $B \rightarrow 0$, the spectrum in Fig. 3 goes over to $\delta(f - \beta_2 f_0)$ and (40) becomes

$$\tilde{R}_d(0, \lambda, 0)_{B \rightarrow 0} = \text{sinc}(\beta_2 f_0 \lambda T), \quad (41)$$

while at the other extreme of a low-pass block spectrum of video bandwidth $B = 2f_0$,

$$\tilde{R}_d(0, \lambda, 0)_{B/2=f_0} = \frac{\text{Si}(2\pi x)}{2\pi x}, \quad x = \beta_2 B \lambda T/2. \quad (42)$$

Equations (41) and (42) are plotted in Fig. 4.

We note from (40) that $\tilde{R}_d(0, \lambda, 0)$ is purely real. Thus from Fig. 4 the 3 dB λ -width (λ_3) of either $\tilde{R}_d(0, \lambda, 0)$ for a wide-band correlator, or of $|\tilde{R}_d(0, \lambda, 0)|$ for a narrow-band correlator, satisfies

$$f_0 \beta_2 \lambda_3 = \frac{0.88}{T} \quad \text{for } B \rightarrow 0 \quad (43)$$

and is only about 11 percent smaller for $B \rightarrow 2f_0$, where f_0 is the centroid of the source spectrum. Thus the allowable mismatch (λ) in delay-rate compensation for a peak² correlator loss of at most 3 dB is essentially

$$|\lambda| \leq \frac{0.44}{f_0 \beta_2 T}. \quad (44)$$

To translate this tolerance into that of an allowable frequency difference, we observe that, for the component of the source signal at the centroid frequency $f = f_0$, the instantaneous frequency difference at $t = 0$ between the signals at the input to the multiplier in Fig. 1 is obtained from (15)–(17) as

$$\begin{aligned} \Delta f_0(0) &= f_0 \epsilon'(0) \\ &\approx f_0 (\beta_2 \lambda_c - \Delta'_0) = f_0 \beta_2 \lambda \end{aligned} \quad (45)$$

so that the abscissa of Fig. 4 may be interpreted as

$$x = T \cdot \Delta f_0(0) \quad (46)$$

where $\Delta f_0(0)$ is the uncompensated frequency difference at $t = 0$ at frequency $(\beta_2 f_0)$. The mismatch tolerance specification given by (44) is then

$$|\Delta f_0(0)| \leq \frac{0.44}{T} \quad (47a)$$

which, for 3 dB peak loss, corresponds to a linear phase rotation at frequency $(\beta_2 f_0)$ during time T of

$$2\pi T \cdot \Delta f_0(0) = 2\pi(0.44) \approx 158^\circ. \quad (47b)$$

If no delay-rate compensation is attempted, then $\lambda_c = 0$ in (17) so that

$$\lambda = -\Delta'_0/\beta_2 \quad (48)$$

(40) and the abscissa in Fig. 4 then has a magnitude of

$$x = |T f_0 \Delta'_0|, \quad (49)$$

which from Table I is seen to be

$$\begin{aligned} x &= |T f_0 (\beta_2 - \beta_1)| \\ &= |T f_0 \beta_1 \beta_2 (r'_{01} - r'_{02})/c| \end{aligned} \quad (50a)$$

²This assumes λT is small enough so that the mainlobe responses versus τ of $R_d(\tau, \lambda, 0)$ for the broad-band, and $|\tilde{R}_d(\tau, \lambda, 0)|$ for the narrow-band correlator are unimodal with global maxima at $\tau = 0$. For the block source spectrum, calculations of $\tilde{R}_d(\tau, \lambda, 0)$ show that this requires that $x = |\beta_2 f_0 \lambda T|$ be less than 0.6 to 1.0, with the exact bound depending on $B/2f_0$ and the correlator type. At larger values of x , the mainlobe responses remain even in τ but become bimodal, with a local minimum at $\tau = 0$.

for moving source and fixed receivers

$$= |Tf_0(r'_{01} - r'_{02})/c| \quad (50b)$$

for fixed source and moving receivers.

Since β_1 and β_2 are within 1 percent of unity, (50a) and (50b) are essentially equal.

2) *Comparisons With Published Results* [2], [3], [5], [13]: From (50b), Fig. 4 is seen to be equivalent to Fig. 4 of [3], but with a more general abscissa.

The approach used and the general results obtained³ are identical to those given in [13] for the case where the delay-acceleration mismatch is zero and where real rather than complex signal notation is used. Thus the real part of (18), with $\alpha = 0$ and $\beta_2 \lambda \rightarrow \delta$, is the same as equation (3.10.2-33), p. 111 of [13]; and for the block source spectrum, the further notational changes $\beta_2 f_0 \rightarrow f_c$, $\beta_2 B \rightarrow B$ reveal that our equations (40), (41), and (42) are identical to equations (3.10.2-35, -36, -38), p. 113, *op. cit.*

The equivalent filter, when no delay-rate compensation is used, is given by (28) and (48) and Table I

$$H_d(f, -\Delta'_0/\beta_2, 0) = \text{sinc}(fT\Delta'_0/\beta_2) \\ = \text{sinc}[(1 - \beta_1/\beta_2)fT] \quad (51)$$

which is slightly different, both from the result given in (21) of [5], and also from the result implied by (26b) of [2]. It appears that the difference from [5] stems from the definition of correlator clock time used there, while the difference from [2] rises from an approximation neglecting spectral compression made there [p. 1548, paragraph preceding (26a)].

As a final comparison we note that the "bias" in the time-delay estimate suggested in [2] as a consequence of ignoring delay rate is an artifact which vanishes if we agree that the correlator-delay estimate applies at the *center* of its data window. The peak of $\tilde{R}_d(\tau, \lambda, 0)$ occurs at $\tau = 0$ for *any* λ^4 , so that from (17) and Table I the mean correlator-delay estimate is

$$\tau_{cm} = \frac{\Delta_0}{\beta_2}, \quad (52)$$

which for the moving source/fixed receiver scenario is

$$\tau_{cm} = \left(\frac{\beta_1}{\beta_2} r_{01} - r_{02} \right) / c \\ = \left(\frac{r_{01} - r_{02}}{c} \right) - (1 - \beta_1/\beta_2) \frac{r_{01}}{c} \\ = \left(\frac{r_{01} - r_{02}}{c} \right) - \left(\frac{r'_{01} - r'_{02}}{c} \right) \beta_1 \frac{r_{01}}{c} \\ = \left(\frac{r_{01} - r_{02}}{c} \right) - \left(\frac{r'_{01} - r'_{02}}{c} \right) \tau_{01} \\ \cong [r_1(-\tau_{01}) - r_2(-\tau_{01})] / c \quad (53)$$

if we neglect the acceleration terms $r''_{0k}\tau_{01}^2/2c$.

³We thank a reviewer for pointing this out.

⁴Assuming as before that λT is not so large that the mainlobe of $\tilde{R}_d(\tau, \lambda, 0)$ becomes bimodal.

Thus in this case the mean correlator *time*-delay difference estimate for *any* λ is a measure of the *range*-difference delay at a time earlier than $t = 0$ by the propagation time (at $t = 0$) from the source to sensor "1." This geometric, or kinematic, bias is a time-lag effect arising from finite propagation velocity, and is almost always negligible.

The corresponding result for the fixed-source/moving-receiver scenario is

$$\tau_{cm} = (r_{01} - r_{02})/c\beta_2 \quad (54)$$

which has a scale-factor geometric error of up to 1 percent, again almost always negligible.

C. Delay-Acceleration Mismatch with Perfect Delay-Rate Match

1) *Derivation of Results:* From (18) with $\lambda = 0$, we have

$$\tilde{R}_d(\tau, 0, \alpha) = \frac{1}{T} \int_{-T/2}^{T/2} R_s[\beta_2(\tau + \alpha t^2/2)] dt \quad (55)$$

with an equivalent expression given by (21) and (29) as

$$\tilde{R}_d(\tau, 0, \alpha) = \int_0^\infty \frac{2}{\beta_2} W_s\left(\frac{f}{\beta_2}\right) \frac{Z(T\sqrt{f\alpha/2})}{T\sqrt{f\alpha/2}} e^{j2\pi f\tau} df \quad (56)$$

where Z is the complex Fresnel integral (25).

In this case both $|\tilde{R}_d(\tau, 0, \alpha)|$ and $\text{Re}[\tilde{R}_d(\tau, 0, \alpha)]$ have their peaks at (different) nonzero values of τ which vary with α

$$\max_{\{\tau\}} |\tilde{R}_d(\tau, 0, \alpha)| = |\tilde{R}_d(\tau_e(\alpha), 0, \alpha)| \quad (57)$$

$$\max_{\{\tau\}} \text{Re}[\tilde{R}_d(\tau, 0, \alpha)] = \text{Re}[\tilde{R}_d(\tau_r(\alpha), 0, \alpha)] \quad (58)$$

so that the peak loss must be evaluated at either τ_e or τ_r , rather than at $\tau = 0$. These α -varying τ -biases arise because the time shift function in the argument of the integrand in (55) is not odd about zero. As a consequence the equivalent filter (in the sense of Fig. 2) has a nonlinear phase response over frequency, the study of which allows a good understanding of these effects.

From (25) and (29), the equivalent filter is

$$H_d(f, 0, \alpha) = \frac{C(x) + jS(x)}{x}, \quad x \triangleq T\sqrt{\frac{f\alpha}{2}} \quad (59)$$

which we write in amplitude/phase response form as

$$H_d(f, 0, \alpha) = A(f, 0, \alpha) e^{j\phi(f, 0, \alpha)} \quad (60)$$

where

$$A(f, 0, \alpha) = \frac{1}{x} \sqrt{C^2(x) + S^2(x)} \quad (61)$$

$$\phi(f, 0, \alpha) = \tan^{-1} [S(x)/C(x)]. \quad (62)$$

The phase-delay (T_ϕ) and group-delay (T_g) functions for this filter are [10]

$$T_\phi(f) = \frac{-\phi(f, 0, \alpha)}{2\pi f} = \frac{-\alpha T^2}{4\pi x^2} \tan^{-1} [S(x)/C(x)] \quad (63)$$

$$T_g(f) = \frac{-1}{2\pi} \frac{\partial \phi(f, 0, \alpha)}{\partial f}$$

$$= \frac{-\alpha T^2}{8\pi x} \left[\frac{C(x) \sin(\pi/2)x^2 - S(x) \cos(\pi/2)x^2}{C^2(x) + S^2(x)} \right], \quad (64)$$

with limiting small-argument values given by

$$\lim_{x \rightarrow 0} [T_\phi(f)] = \lim_{x \rightarrow 0} [T_g(f)] = \frac{-\alpha T^2}{24} \quad (65)$$

which is the average of the function $(-\alpha t^2/2)$ over the interval $(-T/2, T/2)$.

The amplitude, phase, and delay responses are plotted in Fig. 5 versus the scaled frequency variable $x^2 = f\alpha T^2/2$.

Now consider the block source spectrum of width B centered at f_0 . The frequency domain relations pertinent to (56) are similar to Fig. 3, but with $\text{sinc}(\lambda T f)$ replaced by $H_d(f, 0, \alpha)$.

For a very narrow-band signal ($B \rightarrow 0$), the compressed source spectrum goes over to $\delta(f - \beta_2 f_0)$, and (56) gives

$$\tilde{R}_d(\tau, 0, \alpha) \Big|_{B=0} = e^{j2\pi f_0 \beta_2 \tau} \left[\frac{C(y_0) + jS(y_0)}{y_0} \right]$$

where $y_0 = T \sqrt{\frac{f_0 \beta_2 \alpha}{2}}$. (66)

Here the envelope of the mean correlator output is

$$|\tilde{R}_d(\tau, 0, \alpha)| = \frac{1}{y_0} \sqrt{C^2(y_0) + S^2(y_0)} \quad (67)$$

which is independent of τ since $B = 0$, while the real part of the mean correlator output is

$$\text{Re}[\tilde{R}_d(\tau, 0, \alpha)] = \frac{C(y_0) \cos 2\pi f_0 \beta_2 \tau - S(y_0) \sin 2\pi f_0 \beta_2 \tau}{y_0}$$

$$= \frac{1}{y_0} \sqrt{C^2(y_0) + S^2(y_0)} \cdot \cos 2\pi f_0 \beta_2 [\tau - T_\phi(f_0 \beta_2)] \quad (68)$$

where $T_\phi(f_0 \beta_2)$ is the phase delay of the equivalent filter at frequency $(f_0 \beta_2)$, as given by (63) with $x = y_0$. From (65), (66), and (68) and Fig. 5, we see that for

$$y_0^2 = f_0 \beta_2 \alpha T^2 / 2 \leq 1.73 \quad (69)$$

then both the envelope and the peak real response of the correlator output are attenuated by less than 3 dB, while

$$T_\phi(f_0 \beta_2) \approx \frac{-\alpha T^2}{24}. \quad (70)$$

Thus, for a very narrow-band signal with centroid frequency f_0 , the 3 dB tolerance value for delay-acceleration mismatch is, from (69),

$$|\alpha| \leq \frac{3.46}{\beta_2 f_0 T^2}. \quad (71)$$

For the component of the source signal at its centroid frequency f_0 , the difference-frequency rate between the signals

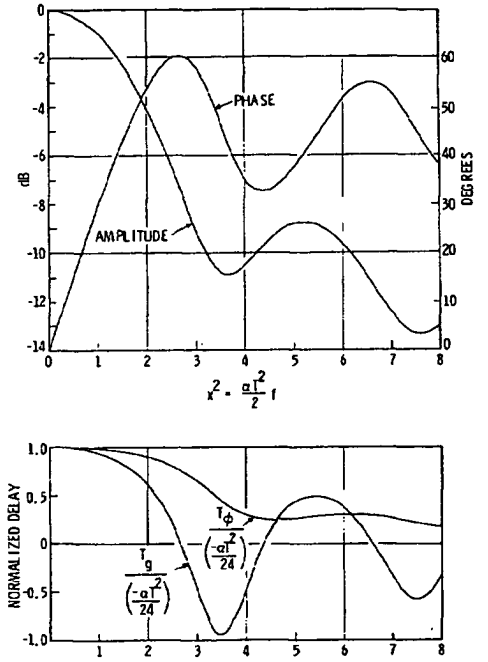


Fig. 5. Amplitude, phase, phase-delay, and group-delay functions for

$$H_d(f, 0, \alpha) = \frac{C\left(T\sqrt{\frac{f\alpha}{2}}\right) + jS\left(T\sqrt{\frac{f\alpha}{2}}\right)}{T\sqrt{\frac{f\alpha}{2}}}$$

at the input to the multiplier in Fig. 1 is obtained from (15)-(17) as

$$\Delta f'_0(0) = f_0 \epsilon''(0) \approx f_0 \beta_2 \alpha \quad (72)$$

so that the equivalent 3 dB tolerance for a mismatch in difference-frequency rate is

$$|\Delta f'_0(0)| \leq \frac{3.46}{T^2} \quad (73)$$

which, at the allowable maximum for 3 dB peak loss, corresponds to a quadratic phase rotation at the edge of the correlator window of

$$\Delta \phi_{\text{edge}} = 2\pi \int_0^{T/2} \Delta f'_0(0) \cdot t \, dt$$

$$= \frac{\pi}{2} \Delta f'_0(0) \cdot T^2/2$$

$$= \frac{\pi}{2} (1.73) \approx 156^\circ \quad (74)$$

where $\Delta f'_0$ is the difference-frequency rate at frequency $\beta_2 f_0$.

If now we allow the signal bandwidth B to increase from zero, then the input to the equivalent filter of Fig. 2 is given by (39), which has the form of a "carrier" at frequency $(f_0 \beta_2)$, modulated by $\text{sinc}(\beta_2 B \tau)$. If $\mu = (B/2f_0)$ is small enough that $H_d(f, 0, \alpha)$ has essentially constant amplitude and linear phase over the band $(\beta_2 B)$ centered at $(\beta_2 f_0)$, then [10] the output envelope is delayed by $T_g(\beta_2 f_0)$, while the

output "carrier" is delayed by $T_\phi(\beta_2 f_0)$, which is the delay for the peak of $\text{Re}\{\tilde{R}_d(\tau, 0, \alpha)\}$. In either case, the output peak is approximately reduced by the "narrow-band" factor given by (67).

As $\mu = (B/2f_0)$ increases further, the concepts of phase delay and group delay lose meaning [10], and we must evaluate both $|\tilde{R}_d|$ and $\text{Re}\{\tilde{R}_d\}$ versus τ to find first the τ -biases, and then the peak attenuations at these values of τ . For the block spectrum this requires numerical integrations which are simpler if we use (38) and (55) to obtain

$$\begin{aligned}\tilde{R}_d(\tau, 0, \alpha) &= \frac{1}{T} \int_{-T/2}^{T/2} \frac{\sin \pi B \beta_2 (\tau + \alpha t^2/2)}{\pi B \beta_2 (\tau + \alpha t^2/2)} \\ &\quad \cdot e^{j2\pi f_0 \beta_2 (\tau + \alpha t^2/2)} dt \\ &= 2 \int_0^{1/2} \frac{\sin 2\pi \mu (x + a\xi^2)}{2\pi \mu (x + a\xi^2)} e^{j2\pi (x + a\xi^2)} d\xi \quad (75)\end{aligned}$$

with

$$\xi = t/T$$

$$x = f_0 \beta_2 \tau$$

$$a = f_0 \beta_2 \alpha T^2/2$$

$$\mu = B/2f_0.$$

Equation (75) was evaluated numerically for various μ and a in the region around $x = -a/12$ (i.e., $\tau = -(\alpha T^2/24)$), and the peak delays and corresponding peak responses obtained for both $\text{Re}\{\tilde{R}_d(\tau_r, 0, \alpha)\}$ and $|\tilde{R}_d(\tau_e, 0, \alpha)|$. The results are shown in Fig. 6, in which the results for $B = 0$ are simply the values given in Fig. 5 for $f = f_0 \beta_2$. It is seen that the delay-acceleration mismatch tolerance for a 3 dB peak loss is very nearly independent of bandwidth for both wide-band and narrow-band correlators, and is essentially given by (71) or (73).

We note from (72) and (73) that the abscissa in Fig. 6 may be expressed as

$$a = \Delta f_0'(0) \cdot T^2/2 \quad (76)$$

and that if no delay-acceleration compensation is attempted, then $\alpha_c = 0$ in (17) and

$$\alpha = -\Delta_0''/\beta_2 \quad (77)$$

so that

$$|a| = |f_0 \Delta_0'' T^2/2| \quad (78)$$

and the algebraic signs of τ_r and τ_e are reversed in Fig. 6. From Table I, (78) is seen to be

$$|a| = \left| f_0 \frac{T^2}{2} (\beta_1^3 r_{01}'' - \beta_2^3 r_{02}'')/c \right| \quad (79a)$$

for moving source/fixed receivers

$$= \left| f_0 \frac{T^2}{2} (r_{01}'' - r_{02}'')/c \right|$$

for fixed source/moving receivers

and for $|1 - \beta_k| < 0.01$, these are roughly equal.

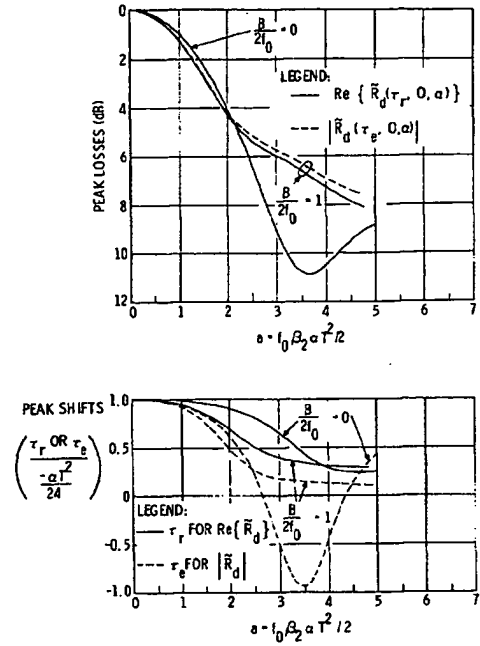


Fig. 6. Peak shifts and losses for delay-acceleration mismatch with wide-band $[\text{Re}(\tilde{R}_d)]$ and narrow-band $[\tilde{R}_d]$ correlators (block source spectrum, bandwidth B , centroid f_0).

2) *Comparisons with Published Results* [3], [4]: For the narrow-band ($B/2f_0 \rightarrow 0$) case, the peak loss for no delay-acceleration compensation given in (4-32) of [4] is essentially (68) with

$$y_0 = T\sqrt{-f_0 \Delta_0''/2} = jT\sqrt{f_0 \Delta_0''/2} \quad (80)$$

so that by (26), the sign of the phase delay $T_\phi(f_0 \beta_2)$ in (68) is reversed. In [4] the small-argument approximation (70) is used.

In (44) of [3] the degradation factor for $B/2f_0 \rightarrow 0$ is given as $C(y_0)/y_0$, which is erroneous since from (68) this equals $\text{Re}[\tilde{R}_d(0, 0, \alpha)]$, which is neither the real output peak nor the peak of the envelope of the output. This error appears to have arisen because the term involving the Fresnel sine integral was ignored in passing from (43) to (44), *op. cit.*

These results are plotted in Fig. 7 for comparisons.

IV. SUMMARY

We have considered the problem of how, and how well, to compensate for the effects of source or receiver motions on the output of a cross correlator used to estimate delay difference across a baseline. For a quadratic delay difference $\Delta(t)$ during the correlator integration period T given by

$$\Delta(t) = \Delta_0 + \Delta_0' t + \Delta_0'' t^2/2, \quad |t| \leq T/2, \quad (4)$$

the required correlator compensation was found to be essentially a quadratic delay modulation $\Delta_c(t)$ inserted in one input channel before multiplication

$$\Delta_c(t) = \tau_c + \lambda_c t + \alpha_c t^2/2, \quad |t| \leq T/2. \quad (6)$$

The approach used was to assume a stochastic source signal and to determine the mean (ensemble average) correlator out-

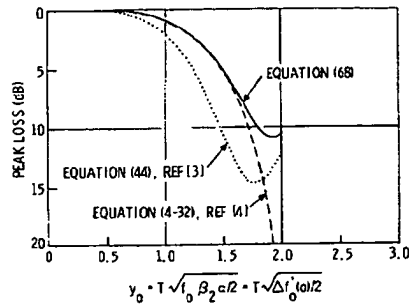


Fig. 7. Comparison of results for peak output loss of correlator with quadratic-delay mismatch and negligible signal bandwidth.

$$\left(\frac{B}{2f_0} = 0 \right).$$

put as a function of the mismatches

$$\begin{aligned} \lambda &= \lambda_c - \Delta'_0/\beta_2 \\ \alpha &= \alpha_c - \Delta''_0/\beta_2, \quad \beta_2 = 1 - \tau'_{02} \end{aligned} \quad (17)$$

in compensating for delay-difference rate and acceleration. The use of analytic signal representation allowed broad-band and narrow-band correlators to be treated simultaneously.

For no more than 3 dB reduction in the peak of the mean correlator output with a *narrow-band* signal and a perfect delay-acceleration match ($\alpha = 0$), the delay-rate mismatch (λ) must satisfy

$$|\lambda T| \leq \frac{0.44}{\beta_2 f_0} \quad \text{or} \quad |T \Delta f'_0(0)| \leq 0.44, \quad (44, 47)$$

while with a perfect delay-rate match ($\lambda = 0$), the delay-acceleration mismatch (α) must satisfy

$$|\alpha T^2/2| \leq \frac{1.73}{\beta_2 f_0} \quad \text{or} \quad \left| \frac{T^2}{2} \Delta f''_0(0) \right| \leq 1.73 \quad (71, 73)$$

where f_0 is the centroid frequency of the source spectrum, T is the correlator integration time, and $\Delta f_0(0)$, $\Delta f'_0(0)$ are the frequency difference and frequency-difference rate at frequency $\beta_2 f_0$ and $t=0$, at the input to the correlator multiplier. For *broad-band* signals, these mismatch tolerances become about 11 percent tighter.

For given values of λ and α , these mismatch tolerances prescribe the maximum allowable correlator integration time T_{\max} as the smaller of

$$T_{m\lambda} = \frac{0.44}{\beta_2 f_0 |\lambda|}$$

and

$$T_{m\alpha} = \sqrt{\frac{3.46}{\beta_2 f_0 |\alpha|}}.$$

If the correlator does not attempt to compensate for delay-difference accelerations, then $\alpha_c = 0$ and α is determined by the kinematics Δ''_0 and may be large, leading to a small value of T_{\max} . A similar conclusion holds for a correlator which does not compensate for delay-difference rates.

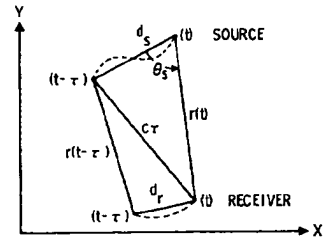


Fig. 8. Assumed propagation geometry.

APPENDIX A

SIGNAL RANGE-DELAY VARIATION WITH MOVING SOURCE OR RECEIVER

The aim here is to describe the time-varying signal delay $\tau = \tau(t)$ in terms of the time-varying range $r(t)$ when the source or receiver, or both, move. Assuming direct-path propagation in a horizontal plane in an idealized medium with constant speed of propagation c , the geometry is indicated in Fig. 8.

The source transmits a waveform $s(t - \tau)$ at time $(t - \tau)$ when the range is $r(t - \tau)$. Neglecting attenuation and distortion in the medium, this point value of the source waveform travels along the path $c\tau(t)$ and impinges on the receiver at time t when the range is $r(t)$, so that the received waveform at time t is

$$s_r(t) = s(t - \tau(t)). \quad (A1)$$

For a fixed source and moving receiver (FS, MR) then $d_s = 0$ in Fig. 8, and

$$c\tau(t) = r(t). \quad (A2)$$

For a moving source and fixed receiver (MS, FR) then $d_r = 0$ so that

$$c\tau(t) = r(t - \tau(t)). \quad (A3)$$

When both source and receiver move (MS, MR), then by the law of cosines

$$c\tau(t) = r(t) \sqrt{1 - 2d_s \cos \theta_s / r(t) + (d_s/r(t))^2} \quad (A4)$$

where $d_s = d_s(t, \tau)$ is the straight line connecting the source positions at times $(t - \tau)$ and (t) , and $\theta_s = \theta_s(t, \tau)$ is the angle as indicated between d_s and $r(t)$. With an instantaneous source velocity schedule given by $v_{sx}(t)$, $v_{sy}(t)$ during $(t - \tau, t)$, then the average source speed over this interval is

$$V_s = \sqrt{V_x^2 + V_y^2} = V_s(t, \tau) \quad (A5)$$

with

$$V_x = \frac{1}{\tau} \int_{t-\tau}^t v_{sx}(\xi) d\xi = V_x(t, \tau) \quad (A6)$$

and similarly for V_y . The length d_s is then

$$d_s = V_s \cdot \tau(t) \quad (A7)$$

where

$$\tau(t) \approx \frac{r(t)}{c} \quad (A8)$$

so that

$$\frac{d_s}{r(t)} \approx \frac{V_s}{c} < 0.01 \quad (\text{A9})$$

and a first-order expansion of (A4) yields

$$c\tau(t) \approx r(t) - (V_s \cos \theta_s) \tau(t). \quad (\text{A10})$$

Equations (A2), (A3), and (A10) relate $\tau(t)$ to $r(t)$ for the (FS, MR), (MS, FR), and (MS, MR) cases. We now concentrate on the first two cases on the assumption that the range $r(t)$ varies quadratically with time over the observation interval

$$r(t) = r_0 + r'_0 t + \frac{r''_0}{2} t^2, \quad |t| \leq T_{\text{obs}}/2. \quad (\text{A11})$$

1) FS, MR—From (A2) and (A11),

$$\tau(t) = \frac{r(t)}{c} = \tau_0 + \tau'_0 t + \tau''_0 \frac{t^2}{2} \quad (\text{A12})$$

where

$$\begin{aligned} \tau_0 &= \frac{r_0}{c} \\ \tau'_0 &= \frac{r'_0}{c} \\ \tau''_0 &= \frac{r''_0}{c}. \end{aligned} \quad (\text{A13})$$

2) MS, FR—From (A3) and (A11),

$$c\tau = r(t - \tau) = r - \tau r' + \frac{\tau^2}{2} r'' \quad (\text{A14})$$

where we temporarily suppress the explicit time-function notation. The solution of (A14) for τ yields

$$\tau = \frac{(c + r')}{r''} \left[1 - \sqrt{1 - \frac{2r''r}{(c + r')^2}} \right] \approx \frac{r}{c + r'} \quad (\text{A15})$$

since the second term under the radical in (A15) is of the order of c^{-1} times the change in r' during τ , and is thus much less than 1.

Successive differentiation of (A14) yields

$$\begin{aligned} \tau' &= \frac{\frac{r' - \tau r''}{c}}{1 + \left(\frac{r' - \tau r''}{c} \right)} \\ \tau'' &= \frac{\frac{r''}{c} (1 - \tau')}{\left[1 + \left(\frac{r' - \tau r''}{c} \right) \right]^2} = \frac{\frac{r''}{c}}{\left[1 + \left(\frac{r' - \tau r''}{c} \right) \right]^3}. \end{aligned} \quad (\text{A16}) \quad (\text{A17})$$

The signal delay function for the (MS, FR) case is then approximately quadratic for a quadratic range variation

$$\tau(t) \approx \tau_0 + \tau'_0 t + \frac{\tau''_0}{2} t^2 \quad (\text{A18})$$

where from (A11), (A15), \dots , (A17),

$$\begin{aligned} \tau_0 &\approx \frac{r_0/c}{1 + r'_0/c} \\ \tau'_0 &= \left(\frac{r'_0 - r''_0 \tau_0}{c} \right) \left/ \left[1 + \left(\frac{r'_0 - r''_0 \tau_0}{c} \right) \right] \right. \approx \frac{r'_0/c}{1 + r'_0/c} \\ \tau''_0 &= \frac{\frac{r''_0/c}{\left[1 + \left(\frac{r'_0 - r''_0 \tau_0}{c} \right) \right]^3}}{\left[1 + \left(\frac{r'_0 - r''_0 \tau_0}{c} \right) \right]^3} \approx \frac{r''_0/c}{[1 + r'_0/c]^3} \end{aligned} \quad (\text{A19})$$

and where we have assumed that

$$r'_0 - r''_0 \tau_0 = r'(-\tau_0) \approx r'(0) = r'_0.$$

Equations (A13) and (A19) relate the coefficients of the quadratic signal-delay function to the coefficients of the quadratic range function for the two scenarios (FS, MR) and (MS, FR). Note that if $r'' = 0$ all approximations become exact for the (MS, FR) case.

APPENDIX B

DELAY-DIFFERENCE COMPENSATION APPROXIMATIONS

With $\tau_k(t)$ and $\Delta(t) = \tau_1(t) - \tau_2(t)$ assumed quadratic in time (3), (4), then for an *arbitrary* delay-compensation modulation $\Delta_c(t)$, the delay-difference function at the input to the multiplier in Fig. 1 is the negative of $\epsilon(t)$, as given by (15). Combining (3), (4), and (15) gives

$$\epsilon(t) = \frac{\tau''_{02}}{2} \Delta_c^2(t) + [1 - \tau'_{02} - \tau''_{02} t] \Delta_c(t) - \Delta(t). \quad (\text{B1})$$

For perfect compensation we require $\epsilon(t) = 0$ over $(-T/2, T/2)$, so that $\Delta_c(t)$ is the root of a quadratic

$$\begin{aligned} \Delta_c(t) &= \frac{-(\beta_2 - \tau''_{02} t)}{\tau'_{02}} \left[1 - \sqrt{1 + \frac{2\tau''_{02} \Delta(t)}{(\beta_2 - \tau''_{02} t)^2}} \right], \\ \beta_2 &\triangleq 1 - \tau'_{02}. \end{aligned} \quad (\text{B2})$$

In nearly all practical cases the second term under the radical in (B2) is $\ll 1$, so that

$$\Delta_c(t) \approx \frac{\Delta(t)}{\beta_2 - \tau''_{02} t} \approx \frac{\Delta(t)}{\beta_2}, \quad (\text{B3})$$

therefore, if $\Delta(t)$ is quadratic in time, then the optimum $\Delta_c(t)$ is very nearly quadratic in time.⁵ Thus we assume that $\Delta_c(t)$ has the quadratic form given by (6), and when (3), (4), (6), (15) are combined, we obtain

$$\epsilon(t) = A + Bt + \frac{D}{2} t^2 + Et^3 + Ft^4 \quad (\text{B4})$$

where

$$\begin{aligned} A &= \beta_2 \tau_c - \Delta_0 + [\tau''_{02} \tau_c^2/2] \\ B &= \beta_2 \lambda_c - \Delta'_0 - [\tau''_{02} (1 - \lambda_c) \tau_c] \\ D &= \beta_2 \alpha_c - \Delta''_0 - [\tau''_{02} \{ \lambda_c (2 - \lambda_c) - \tau_c \alpha_c \}] \\ E &= [-\tau''_{02} (1 - \lambda_c) \alpha_c/2] \\ F &= [\tau''_{02} \alpha_c^2/8]. \end{aligned} \quad (\text{B5})$$

⁵ This is exact if $\tau''_{02} = 0$.

By neglecting the terms in brackets in (B5), we obtain (16). In any particular scenario the effect of these approximations can be estimated by expanding $\tilde{R}_s[\epsilon(t)]$ to low order in $\epsilon(t)$ and integrating (14). In nearly all cases the approximations leading from (B5) to (16) are very good.

ACKNOWLEDGMENT

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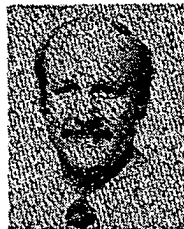


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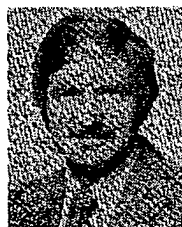
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